

**QP Code :14675**

**(3 Hours)**

**[ Total Marks : 80**

- N.B :** (1) Question No. 1 is **compulsory**.  
(2) Solve any **three** questions out of remaining **three** questions.  
(3) All questions carry **equal** marks as indicated by **figures** to the **right**.  
(4) Assume appropriate data whenever required. State all assumptions clearly.

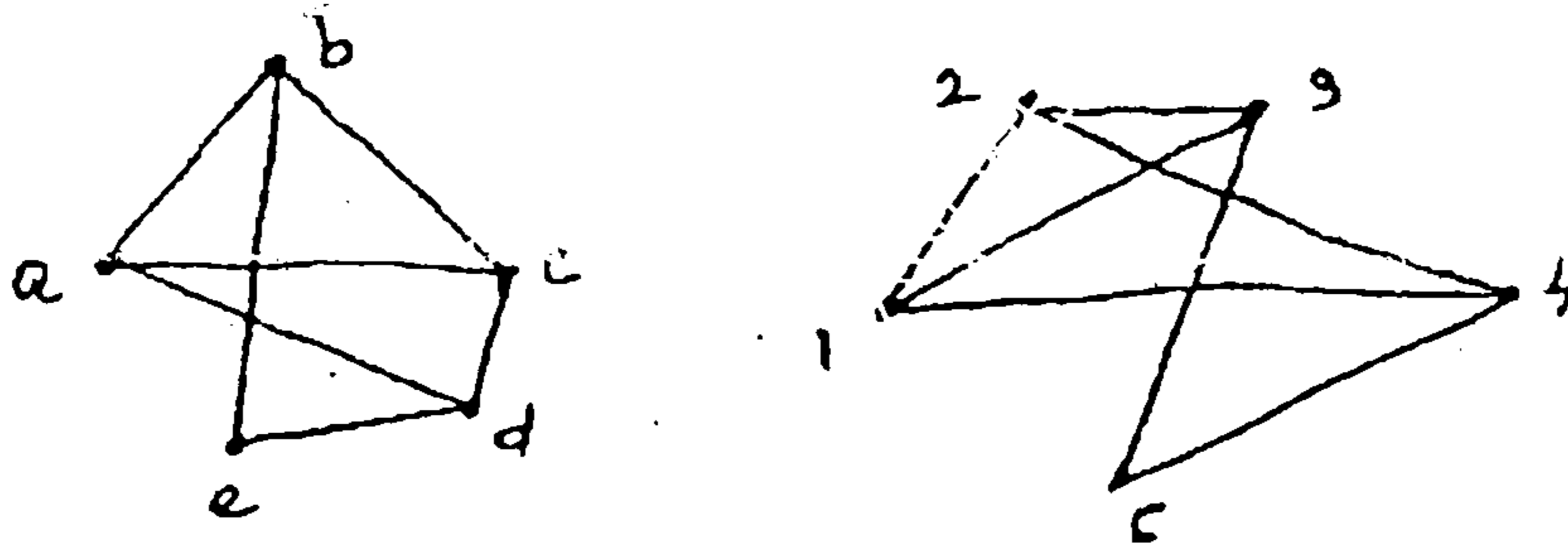
1. (a) Prove by mathematical induction  $x^n - y^n$  is divisible by  $x - y$ . 5  
(b) How many vertices are necessary to construct a graph with exactly 6 edges in which each vertex is of degree 2. 5  
(c) Show that a relation is reflexive and circular if and only if it is an equivalence relation. 5  
(d) Prove that the set  $G = \{1, 2, 3, 4, 5, 6\}$  is an abelian group under multiplication modulo 7. 5
2. (a) Is it possible to draw a tree with five vertices having degrees 1, 1, 2, 2, 4? 4  
(b) Find how many integers between 1 and 60 are 8  
(i) not divisible by 2 nor by 3 and nor by 5. 44  
(ii) Divisible by 2 but not by 3 and nor by 5.  
(c) Solve the recurrence relation  $a_{r+2} - a_{r+1} - 6a_r = 4$  8
3. (a) Show that  $A \cap (B \oplus C) = (A \cap B) \oplus (A \cap C)$  4  
(b) State and explain Pigeonhole principle, extended Pigeonhole principle. How many numbers must be selected from the set  $\{1, 2, 3, 4, 5, 6\}$  to guarantee that at least one pair of these numbers add up to 7? 8  
(c) Let  $R$  be a relation on set  $A = \{1, 2, 3, 4\}$ , given as 8  
 $R = \{(1, 1), (1, 4), (2, 2), (2, 3), (3, 2), (3, 3), (4, 1), (4, 4)\}$ .  
Find transitive closure using Warshall's Algorithm.
4. (a) Find the generating function for the following sequence 4  
(i) 1, 2, 3, 4, 5, 6.....  
(ii) 3, 3, 3, 3, 3.....  
(b) Show that the (2, 5) encoding function  $e: B^2 \rightarrow B^5$  defined by 8  
 $e(00) = 00000$      $e(01) = 01110$   
 $e(10) = 10101$      $e(11) = 11011$  is a group code.  
How many errors will it detect and correct.  
(c) Draw Hasse Diagram of  $D_{42}$ . Find the complement of each element in  $D_{42}$ . 8

5. (a) Define Distributive Lattice along with one appropriate example. 4
- (b) Let the functions  $f, g,$  and  $h$  defined as follows : 8
- $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = 2x+3$
- $g: \mathbb{R} \rightarrow \mathbb{R}, g(x) = 3x+4$
- $h: \mathbb{R} \rightarrow \mathbb{R}, h(x) = 4x$
- Find  $gof, fog, foh, hof, gofoh$

(c) let  $H = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$  8

Be a parity check matrix. Determine the group code  $e_H: B^3 \rightarrow B^6$

6. (a) Determine if  $[(p \Rightarrow q) \wedge \neg q] \Rightarrow \neg p$  is a tautology. 4
- (b) Define isomorphic graphs. Show that following graphs are isomorphic. 8



- (c)  $R$  be a relation on set of integers  $Z$  defined by 8
- $R = \{(x, y) \mid x-y \text{ is divisible by } 3\}$
- Show that  $R$  is an equivalence relation and describe the equivalence classes.